



Patent Office
Canberra

I, KIM MARSHALL, MANAGER PATENT OPERATIONS hereby certify that annexed is a true copy of the Provisional specification in connection with Application No. PP 6342 for a patent by CANON KABUSHIKI KAISHA filed on 02 October 1998.

I further certify that pursuant to the provisions of Section 38(1) of the Patents Act 1990 two Complete specifications were filed on 01 October 1999 and are associated Applications to Provisional Application No. PP 6342 and have been allocated No's 52618/99 and 52620/99.

WITNESS my hand this
Twelfth day of October 1999

A handwritten signature in cursive script, appearing to read "Kim Marshall".

KIM MARSHALL
MANAGER PATENT OPERATIONS



CERTIFIED COPY OF
PRIORITY DOCUMENT

ORIGINAL

AUSTRALIA

Patents Act 1990

PROVISIONAL SPECIFICATION FOR THE INVENTION ENTITLED:

Method and Apparatus for Segmenting Images

Name and Address
of Applicant: Canon Kabushiki Kaisha, Incorporated in Japan, of 30-2,
Shimomaruko 3-chome, Ohta-ku, Tokyo, 146, JAPAN

Name of Inventor(s) Mathieu Hitter, Delphine Anh Dao Le

This invention is best described in the following statement:

METHOD AND APPARATUS FOR SEGMENTING IMAGES

Field of Invention

The present invention relates to the field of image segmentation. In particular, the invention relates to a method and apparatus for segmenting an image. The invention also relates to a computer program product including a computer readable medium having recorded thereon a computer program for segmenting an image.

Background of Invention

Image segmentation is an initial step in many image processing tasks such as pattern recognition, image coding and image interpretation. For example, in scene understanding applications, the segmentation process generally provides a labelling process with regions to be classified.

The publication entitled "Seeded Region Growing" IEEE Trans. Pattern Anal. Machine Intell., vol. 16 pp. 641-647, 1994 (hereinafter called Adams et al) discloses a method for segmentation of images. This method is based on a region growing principle of selecting a pixel adjacent to a region of pixels, which is most similar to the region of pixels. The method does not rely on the arbitrary selection of homogeneity thresholds, but is controlled by choosing a small number of pixels, called seeds. This seed selection may be either automatic or manual. Once the seeds have been selected, the segmented regions are grown in an iterative fashion. Each step of the process involves the addition of one of the neighboring pixels to one of the regions grown from the seeds. A measure $\delta(x)$ is defined how different each of the neighboring pixels is from that region. The neighboring pixel having the minimum measure $\delta(x)$ is added to the region. Adams et al make use of a sorted list in determining the relevant neighboring pixel to be added. In Adams et al, once a pixel has been added to the list, the $\delta(x)$ measure is never updated. However, the Adams et al method suffers from the disadvantage of slow image segmentation.

Aspects of Invention

- It is an object of the present invention to ameliorate one or more disadvantages of the prior art.

One or more exemplary aspects of the invention are listed below, but are not limited thereto.

According to one aspect of the invention, there is provided a method of segmenting an image including the steps of : (i) generating a list of neighboring pixels to a region of pixels; (ii) scanning a limited number of said listed neighboring pixels in a predetermined manner; (iii) determining a value, for each said scanned pixel, indicative

of the similarity of the scanned pixel with the region of pixels; (iv) selecting a pixel from the scanned pixels that has a minimum value; (v) adding said selected pixel to said region; (vi) adding new neighbors of said region to said list; and (vii) repeating steps (ii) to (vi) until the image is segmented.

5 According to another aspect of the invention, there is provided a method of segmenting an image including the steps of : (i) generating a list of neighboring pixels to a region of pixels; (ii) scanning a limited number of said listed neighboring pixels in a predetermined manner; (iii) determining a value, for each said scanned pixel, indicative of the similarity of the scanned pixel with the region of pixels; (iv) selecting the first
10 pixel from the scanned pixels that has a said value below a threshold; (v) adding said selected pixel to said region; (vi) adding new neighbors of said region to said list; and (vii) repeating steps (ii) to (vi) until the image is segmented.

 According to another aspect of the invention, there is provided an apparatus for implementing any one of the abovementioned methods.

15 According to another aspect of the invention, there is provided a computer program product, including a computer readable medium having recorded thereon a computer program for implementing any one of the abovementioned methods.

Brief Description of the Drawings

20 Embodiments of the invention are described with reference to the drawings, in which:

 Fig. 1 is a flow chart of a method of segmenting an image in accordance with a preferred embodiment;

 Fig 2A & 2B are is illustrated an example of a segmented image and a
25 corresponding region adjacency graph;

 Fig. 3 is a flow chart of a method of segmenting an image as used in Fig. 1;

 Fig. 4A is a flow chart of a method of seeding an image as used in Fig. 3;

 Fig. 4B, illustrates an example of a seeded image seeded according to Fig.
4A;

30 Fig. 5A is a flow chart of a method of seeded region growing as used in Fig. 3;

 Fig. 5B illustrates a simplified example of the preferred region growing process of Fig. 5A;

 Fig. 6 is a flow chart of a method of coding a segmented image as used in Fig.
35 1;

 Fig. 7A is a flow chart of splitting a segmented image as used in Fig. 6;

 Figs. 7B to 7E illustrates an example of splitting a segmented image in accordance with the preferred splitting process of Fig. 7A;

Fig. 8A is a flow chart of merging a segmented image as used in Fig. 7;

Figs. 8B and 8C illustrate an example of merging the split segmented images of Fig. 7E in accordance with the preferred merging process of Fig. 8A;

Fig. 9 is a flow chart of MRF merging the segmented image as used in Fig. 1;
5 and

Fig. 10 is a block diagram of a general purpose computer.

Detailed Description

Where reference is made in any one or more of the accompanying drawings to
10 steps and/or features, which have the same reference numerals, those steps and/or
features have for the purposes of this description the same function(s) and/or
operation(s), unless the contrary intention appears.

The principles of the preferred method have general applicability to the
segmentation of multiple colored or black and white images. The method has been
15 described with reference to a number of specific examples of images and it is not
intended that the invention be limited to such specific examples.

1.0 Overview of Preferred Method

Fig 1 is a flow diagram of a method of segmenting an image in accordance
20 with a preferred embodiment of the invention. The method commences at step 102,
where image data is inputted and any necessary parameters are initialized. The image
data is a pixel-map representation of an original image. The pixel-map includes a
numerical representation of the particular color for each pixel location in a rectangular
array. Any numerical representation of color can be used and can be expressed as a
25 sequence of one or more numbers. Preferably, the image data is inputted on a pixel by
pixel basis in raster image order, i.e. from left to right across a display device and from
top to bottom of the display device in a non-interlaced fashion. Pixel locations at the
pixel grid are represented by an array of row (i) and column (j) specifications. Also,
other information may be inputted at this stage, such as header information. Such
30 header information can include the size of the image, i.e. array size (i x j).

In the next step 104, the inputted image is initially segmented into a number of
regions. In this first stage, over-segmentation is preferred rather than under-
segmentation. Regions, which should not have been separated, can be merged later
during the MRF merging step 110. Whereas regions, which should have been
35 separated, cannot be separated later during the MRF merging step 110. The initial
segmentation step 104 is not only fast but also accurate and produces a limited number
of regions; i.e. the over splitting is moderate. The initial segmented image is outputted
as a pixel-map representation. The preferred initial segmentation process is described in

more detail in the section herein entitled "*1.1 Process of Initial Segmentation*". After step 104, the processing continues at step 106.

In step 106, the initial segmented image is converted to a coded representation. This coded representation needs less memory and provides faster access for feature computation (See Table 1) than the pixel-map of the initial segmented image. The preferred embodiment stores the coded segmented image as a list of rectangles or squares instead of the pixel-map representation. The preferred conversion method is described in more detail in the section herein entitled "*1.2 Process of Encoding Initial Segmented Image*".

After step 106, the processing continues at step 108. In this step 108, the coded initial segmented image is represented as a region adjacency graph (RAG). The initial RAG is constructed from the initial segmentation result. Given an image that is segmented into a set of N disjoint regions such that $R = \{R_i, 1 \leq i \leq N\}$, the RAG is constructed by denoting each region R_i as a node and connecting the regions which share a common boundary. Thus, a neighborhood system is defined on the graph, so that only spatially adjacent regions are neighbors. A clique is a subset of R such that it contains either a single node or several nodes that are all neighbors of each other, and where C is the set of all the cliques.

Turning now to, Figs. 2A and 2B there is illustrated an example of a segmented image 200 and a corresponding region adjacency graph 202. The segmented regions 1,2,3,4,5 and 6 of the image 200 are denoted as respective nodes 1,2,3,4,5 and 6 on the region adjacency graph 202. As can be seen, the couplings between the nodes on the graph 202 represent the common boundaries between the regions.

Returning to Fig. 1, the method continues at step 110 where a Markov Random Field (MRF) merging process is commenced. The RAG and coded initial segmented image are inputted to this step 110. This step 110 merges the segmented images in a systematic approach, wherein the number of segmented regions are reduced by taking into account region uniformity of such features as intensity, color texture etc. Discontinuities can also be taken into account by introducing a boundary process. The output of this merging process is a final segmented image. The preferred MRF merging process is described in more detail in the section herein entitled "*1.3 Markov Random Field (MRF) Merging Process*".

The processing of the image segmentation terminates at step 112, where the final segmented image is outputted.

1.1 Initial Segmentation Process

Fig. 3 is a flow chart of a method of initially segmenting an image as used in step 104 of Fig. 1. The method commences at step 102, where the image data is

inputted.. As mentioned above, the image data is a pixel-map representation of an original image. In the next step 304, a list of pixels is generated which are to be used as seeds for region growing. The automatic selection of an appropriate set of pixels or set of small connected regions, called seeds controls the method of initially segmenting the image. The selection of the set of seeds or small regions is critical for the success of the image segmentation. The preferred process for generating these seeds is described in more detail in the next section, herein entitled "*1.1.0 Process for Selecting Seeds*". In the next step 306, the process takes the generated seeds and grows seeded regions in an iterative fashion. At each iteration, all those pixels of the image that border the growing regions are considered. The pixel that is most similar to a region that it borders is appended to that region. The preferred process for growing the seeded regions is described in more detail in the section herein entitled "*1.1.1 Process for Growing Seeded Regions*". This process continues until all pixels have been allocated to an associated region, resulting in a segmented image. After completion of this step 306, the processing terminates and returns to step 106.

1.1.0 Process for Selecting Seeds

The seed selection process 304 is a simple and fast quad-tree approach, which distributes the seeds over the image, but it allocates fewer seeds in homogeneous areas of the image. The seed selection process address color data by preferably processing the luminance image, i.e. a grey-scale image. The homogeneity is measured by a simple contrast criterion: the difference between the minimum and maximum luminance.

The following pseudocode is illustrative of the method of seeding an image for use in Fig 3.

25

Pseudocode SEED

RECTANGLE	A rectangle, given by (x,y) and (width,height)
RECTANGLE_LIST	FIFO list of rectangles
30 SEED_LIST	List of pixels (seeds)
CONTRAST	Difference between min and max luminance
HI_MIN_SIZE	Maximum block size for contrast assessment
LO_MIN_SIZE	Minimum block size for block splitting (< HI_MIN_SIZE)
HI_DENSITY, LO_DENSITY	Densities for pixel spreading
35 HI_THRESHOLD, LO_THRESHOLD	Contrast thresholds

Initialize RECTANGLE_LIST with the rectangle corresponding to the whole image.

```
while RECTANGLE_LIST is not empty

    remove first element from RECTANGLE_LIST and keep it in RECTANGLE ;
    assess CONTRAST for area of the image corresponding to RECTANGLE ;

5
    if CONTRAST < LO_THRESHOLD
        add the pixel corresponding to the center of RECTANGLE to SEED_LIST ;
        continue loop ;

10
    if RECTANGLE size > HI_MIN_SIZE
        split RECTANGLE into four and add the new rectangles in
        RECTANGLE_LIST ;
        continue loop ;

15
    if CONTRAST < HI_THRESHOLD
        spread pixels over RECTANGLE with LO_DENSITY ;
        add them to SEED_LIST ;
        continue loop ;

20
    if RECTANGLE size > LO_MIN_SIZE
        split RECTANGLE into four and add the new rectangles in
        RECTANGLE_LIST ;
        continue loop ;

25
    spread pixels over RECTANGLE with HI_DENSITY ;
    add them to SEED_LIST ;
```

endwhile

30 Turning now to Fig. 4, there is shown a flow chart of the last mentioned pseudocode called SEED. The seed selection processing commences at step 402, where rectangle co-ordinates corresponding to the entire image are stored in a FIFO buffer called RECTANGLE_LIST. After step 402, the processing continues at decision block 404, where a check is made whether the RECTANGLE_LIST is empty. If the decision

35 block returns true then processing is terminated at step 406. Otherwise, the processing continues at step 408, where the first element in RECTANGLE_LIST is removed and stored in the variable rectangle.

In the next step 410, the contrast of the entire image or a sub-image thereof corresponding to the removed rectangle is determined. The contrast is determined by calculating the difference between the minimum and maximum luminance values of the pixels in the rectangle. After step 410, the processing continues at decision block 412, where a check is made whether the determined contrast is less than a predetermined low threshold value called LO_THRESHOLD. If the decision block 412 returns true, then the co-ordinates of central pixel of the rectangle are added to a list called seed_list. In this way, sub-images corresponding to the rectangle which have a low contrast and which are of any size have a center as a seed (see Table A). If the decision block 412 returns false, the processing continues at decision block 416. In decision block 416, a check is made whether the size of the rectangle is greater than a predetermined constant called HI_MIN_SIZE. If the decision block 416 returns true, then the processing continues at step 418. In step 418, the rectangle is divided into four sub-rectangles in the manner of a quadtree approach. In this way, sub-images corresponding to rectangles of a large size having medium and high contrast are split (see Table A). After step 418, the processing continues at step 404, where the four sub-rectangles are added to the FIFO buffer RECTANGLE_LIST. If however, the decision block 416 returns false, the processing continues at decision block 420.

In the decision block 420, a check is made whether the determined contrast is less than a predetermined high threshold value called HI_THRESHOLD. If the decision block 420 returns true, then processing continues at step 422, where a number of pixels from the rectangle are added to the SEED_LIST as seeds. These newly added seed pixels are evenly distributed throughout the current rectangle in such a manner that there is a low density of such seed pixels in the rectangle. In this way, a low density seeding is achieved for sub-images of a small and medium size (See Table A). After step 422, the processing continues at step 404. If, however, the decision block 420 returns false then the processing continues at decision block 424.

In the decision block 424, a check is made whether the size of the rectangle is greater than a predetermined minimum size called LO_MIN_SIZE. If the decision block returns false, the processing continues at step 426, where a number of pixels from the rectangle are added to the SEED_LIST as seeds. These newly added seed pixels are evenly distributed throughout the current rectangle in such a manner that there is a high density of such seed pixels in the rectangle. In this way, a high density seeding is achieved for sub-images of a small size (See Table A). If, however, the decision block 424 returns true then the processing continues at step 428. In step 428, the rectangle is divided into four sub-rectangles in the manner of a quadtree approach. In this way, corresponding sub-images of a medium size and high contrast are split (see Table A).

TABLE A

Allocation of seeds as a function of :

- 5 (1) the contrast of current sub-image, and
(2) the size of the rectangle corresponding to the current sub-image as compared to the size of the rectangle corresponding to the entire image

	Small size	Medium size	Large size
Low contrast	Center of rectangle is a seed	Center of rectangle is a seed	Center of rectangle is a seed
Med contrast	Low density seeding	Low density seeding	Split rectangle
High contrast	High density seeding	Split rectangle	Split rectangle

Turning now to Table A, it can be seen that the split rectangular regions of the
10 image of any size whose pixels have small variance in luminance (low contrast) are seeded in their center. In addition, split rectangular regions of a small or medium size whose pixels have a medium variance in luminance (medium contrast) are seeded evenly throughout these regions in a low density manner. Furthermore, rectangular regions of a small size whose pixels have a high variance in luminance (high contrast)
15 are seeded evenly throughout the region in a high density manner. On the other hand, rectangular regions of medium size and high contrast are split into four rectangular sub-regions. In addition, rectangular regions of a large size and of a medium or high contrast are also split into rectangular sub-regions. This splitting continues in a quadtree manner until the split sub-region(s) meets the abovementioned relevant size
20 and contrast requirements for seeding.

Turning now to Fig. 4B, there is shown an example of a seeded image 452 seeded according to the preferred process 306. For simplicity's sake, the image itself is not shown. Initially, during the seeding process, the original image is inputted and its contrast is determined. As the original image in this example has a medium contrast and
25 is of a large size (as compared to itself), the image is split into four rectangles 454, 456, 458, and 460. The process then considers each of these rectangles 454, 456, 458, and 460. As the images, in this example, within rectangles 454, 456, and 458 are of low contrast, and of a large size as compared to the original image, the centers of these rectangles are taken as seeds 462. However, as the image, in rectangle
30 460 is of a high contrast and large size, the rectangle is split further into four sub-rectangles 464, 466, 468, and 470. The process then considers each sub-rectangle 464, 466, 468, and 470. As rectangle 464 and 466 are both of a high contrast and

medium size they are each further split into four sub-rectangles. As rectangle 468 is of a medium contrast and size, the rectangle is allocated seeds 472 in a low density manner. In addition, as rectangle 470 is of a low contrast and medium size, the center of this rectangle is taken as a seed 474. The seeding processing continues in a similar manner, until all split rectangles have been seeded. In this particular example, the split rectangles are center seeded 476,478,480,482,484, and 486 and the remaining split rectangles are allocated seeds 488, and 490 in a high density manner. At the completion of the seeding process, a list of all the pixel locations of seeds 462,472,474,476,478,480,482,484,486, 488, and 490 is established.

The preferred seeding process is a simple and fast approach which distributes the seeds over the entire image, while allocating fewer seeds in homogeneous areas of the image. Furthermore, there is a high probability at least one seed will be allocated to each homogeneous region of the image.

1.1.1 Process for Growing Seeded Regions

The seeded region growing process 306 takes a set of seeds, individual pixels or small groups of connected pixels, generated by step 304, as input. The preferred process 306 grows the seed regions in an iterative fashion. At each iteration, all those pixels that border the growing regions are considered. The pixel, which is most similar to a region that it borders, is appended to that region. In the preferred process, all the regions can be grown simultaneously.

The process evolves inductively from the seeds, namely, the initial state of the sets or regions A_1, A_2, \dots, A_n . Each step of the iterative process involves the addition of one pixel to one of the above sets. We now consider the state of the sets (A_i) after m steps. Let T be the set of all as-yet unallocated pixels which border at least one of the regions.

$$T = \left\{ x \notin \bigcup_{i=1}^n A_i \mid N(x) \cap \bigcup_{i=1}^n A_i \neq \emptyset \right\}, \quad \text{Eqn (1)}$$

where $N(x)$ is the set of immediate neighbors of the pixel x . For each candidate pixel x , an index $i(x)$ is found, which correspond to the adjacent region where x is most likely to be included and a criterion $\delta(x)$ is computed; $\delta(x)$ measures how good a candidate x is for region A_i .

If, for $x \in T$ we have that $N(x)$ meets just one of the A_i , then we define $i(x) \in \{1, 2, \dots, n\}$ to be that index such that $N(x) \cap A_{i(x)} \neq \emptyset$ and define $\delta(x)$ to be a measure of how different x is from the region it adjoins. The simplest definition for $\delta(x)$ is

$$\delta(x) = \left| g(x) - \text{mean}_{y \in A_{i(x)}}[g(y)] \right|, \quad \text{Eqn (2)}$$

where $g(x)$ is the grey value of the image point x for grey-scale images. The extension of this criterion to color images requires the choice of a suitable metric in color space. For example, the absolute value of the Euclidean distance between the color of pixel x and the mean color of region $A_i(x)$ could be used. Alternatively, the segmentation
5 could be performed on the luminance image as per on gray-scale images.

If $N(x)$ meets two or more of the A_i , we take $i(x)$ to be a value of i such that $N(x)$ meets A_i and $\delta(x)$ is minimized.

Then, a candidate pixel $z \in T$ is chosen such that

$$\delta(z) = \min_{x \in T} \{\delta(x)\}. \quad \text{Eqn (3)}$$

10 and append z to $A_{i(z)}$.

This completes step $m+1$. The process is repeated until all pixels have been allocated. The process commences with each A_i being just one of the seeds. The equations 2 and 3 ensure that the final segmentation is into regions as homogeneous as possible given the connectivity constraint.

15 In the preferred process 306, $\delta(x)$ is updated only for a limited number of the candidate pixels at each step of the iteration. Consequently, as the color of the limited number of candidate pixels is always compared with the updated mean color of the neighboring regions, the quality of the segmentation is comparable with Adams et al. Furthermore, as the process does not consider all the candidate pixels, especially when
20 the list is relatively long, the speed of the region growing process can be significantly increased without reducing the quality of the final segmentation. The region growing process 306 uses two ways, either alone or in combination, to avoid scanning the whole candidate pixels list.

The first one is to use a variable step when scanning the candidate pixel list,
25 the value of the step depending on the size of the candidate list: the longer the list, the bigger the step. Another advantage of this method is a better control on the processing time (if there is a linear relation between the size of the candidate list and the step).

The second process consists in skipping a whole part of the list by choosing the first candidate pixel x such that $\delta(x)$ is smaller than $\delta(z)$, z being the pixel selected at
30 the previous step. If such a pixel is found, then the scanning of the list is interrupted prematurely; otherwise, the whole list is scanned to find the candidate pixel with the minimum $\delta(x)$ value, and the threshold is tuned to that value.

As successive best candidates often belong to the same region, inserting the new candidates (neighbors of the selected pixel) at the beginning of the list can reduce
35 the computation time. However, they are not considered at the first step after their insertion in order not to introduce a bias in favor of a particular region.

The following pseudocode is illustrative of the preferred method of seeding an image for use in Fig 3.

Pseudo-code REGION GROWING

SEED_LIST	List of seeds (pixels)
CANDIDATE_LIST	List of pixels which are neighboring at least one region
5 REGION []	Array used to store the growing regions i.e. the lists of classified pixels
MEAN []	Array containing the mean gray value or luminance color of the regions
10 DELTA	Function measuring the difference between a pixel and a neighboring region
MIN	Variable used for storing the minimum DELTA
CHOSEN_PIX	Chosen pixel
CHOSEN_REG	Index of the chosen region
DYN_THRESHOLD	Dynamic threshold to allow early selection of a candidate pixel
15 DYN_STEP	Dynamic step for the scan loop of CANDIDATE_LIST
DYN_START	Dynamic starting position for the scan loop

Initialize each REGION with the corresponding seed of SEED_LIST and initialize CANDIDATE_LIST with the neighbors of each seed ;
20 DYN_THRESHOLD = 0 ;
DYN_START = 0 ;

while CANDIDATE_LIST is not empty
 Set DYN_STEP depending the size of CANDIDATE_LIST, e.g. DYN_STEP
25 = size of CANDIDATE_LIST / 300
 for i = DYN_START to size of CANDIDATE_LIST, i = i + DYN_STEP
 CURRENT_PIX = pixel i in CANDIDATE_LIST
 if (DELTA(CURRENT_PIX) < MIN)
 MIN = DELTA(CURRENT_PIX)
30 CHOSEN_PIX = PROCES_PIX
 CHOSEN_REG = index of the chosen region
 stop for if MIN < DYN_THRESHOLD
 endfor
 put each unclassified neighbor of CHOSEN_PIX in CANDIDATE_LIST and set
35 DYN_START as the number of new pixels in CANDIDATE_LIST ;
 Put CHOSEN_PIX in REGION[CHOSEN_REG] ;
 Update MEAN[CHOSEN_REG] ;
 Remove CHOSEN_PIX from CANDIDATE_LIST;

DYN_THRESHOLD = max (DYN_THRESHOLD, MIN)
endwhile

Turning now to Fig. 5A, there is shown a flow chart of the last mentioned pseudocode named REGION GROWING for growing the segmented regions. The processing commences at step 502, where the seed list is stored in an array called REGION[]. This array REGION[] is used to store the growing regions, i.e. the lists of classified pixels. Initially, the seeds of the image are denoted as the initial regions for growing.

In the next step 504, the neighboring pixels of each seed are determined and stored in a list called CANDIDATE_LIST. In the next step 506, a variable DYN_THRESHOLD is set to zero. This variable stores a dynamic threshold to allow early selection of a candidate pixel in a manner, which will be explained below. After step 506 the processing continues at decision box 508, in which a check is made whether the CANDIDATE_LIST is empty. The CANDIDATE_LIST will be empty once there are no more pixels neighboring the growing regions. If the decision box 508 returns true then the processing continues at step 510 where the region growing process 306 is completed. If the decision block 508 returns false then the processing continues at step 512.

In step 512 the variable loop counter i is set to zero, the variable MIN is set to 256, and the variable DYN_STEP is set to the current size of the CANDIDATE_LIST divided by 300. The variable MIN is used for storing the minimum delta value of the previous iteration of the loop 508-512,...,and 536. The variable DYN_STEP is used for storing a variable step value used for scanning the CANDIDATE-LIST. This variable step value is used for determining the delta values for a limited number of candidates in the CANDIDATE-LIST. Specifically, only those candidates spaced apart by a value equal to the step value will be considered for allocation to the region. After step 512, the processing continues at decision box 514, where a check is made whether the loop counter is less than the size of the CANDIDATE_LIST.

If the decision block 514 returns false, the processing continues at step 526, which is described below. If, however, the decision box 514 returns true then the region growing process has not considered all the limited number of neighboring pixels. In this situation the processing continues at step 516, where the variable CURRENT_PIX is set to pixel i in the CANDIDATE_LIST. This step 516 sets the next candidate pixel to be considered. It should be noted that this pixel is spaced apart from the previous pixel considered by a distance equal to the value stored in the variable DYN_STEP. After step 516, the processing continues at the decision box 518.

In decision box 518, a comparison is made whether the difference between the luminance value of the current candidate pixel and the mean of the neighboring region

is less than MIN. If the decision box 518 returns false, then the processing continues at step 520. In step 520 the loop counter *i* is incremented by the step value stored in DYN_STEP. If the decision box 518, returns true, then the processing continues at step 522. In step 522, the MIN variable is now set to the minimum delta value
5 determined for the current pixel. In addition, the variable CHOSEN_PIX is set to the selected current pixel and the variable CHOSEN_REG is set to the index of the current region. After step 522, the processing continues at step 524.

In decision block 524, a comparison is made whether the current minimum delta value stored in MIN is less than the current value stored in DYN_THRESHOLD.
10 If the decision block 524 returns false then the processing continues at step 520, where the loop counter *i* is incremented by the step value stored in DYN_STEP. Otherwise, if the decision block 524 returns true then the processing continues at step 526. In step 526, each pixel neighboring the current pixel stored in CHOSEN_PIX, and not previously stored in the CANDIDATE_LIST, is now added to the
15 CANDIDATE_LIST. After step 528, the processing continues at step 530, where the current pixel stored in CHOSEN_PIX is added to the region which is stored in REGION[CHOSEN_REG]. At the next step 534, the current pixel stored in CHOSEN_PIX is removed from the candidates in CANDIDATE_LIST. The processing then continues at step 5636, where the variable is reset to the maximum value of the
20 current values stored in MIN and DYN_THRESHOLD. After which, the processing returns to decision block 508.

Fig. 5B illustrates a simplified example of the preferred region growing process. For simplicities sake, this example shows only one region of the region growing process, whereas the preferred method allows the simultaneous growing of
25 multiple regions. An initial region 550 consisting of a plurality of pixels (not shown) is surrounded by a number of candidate pixels 552 to be added to the region 550. Firstly, the process calculates the mean of the luminance values of the pixels of the region 550. Then the process determines the difference between this mean and the luminance value of a limited number of candidate pixels 552 in turn. The process then determines the
30 minimum difference of these differences and allocates the candidate pixel associated with this minimum difference to the region 550. If however, the luminance difference value of any candidate pixel is less than the minimum difference value in the previous iteration, then the process instead allocates this candidate pixel to the region 550 and then proceeds to the next iteration. In the next iteration, the mean value of the
35 luminance values of the pixels of the grown region 550 is then recalculated and the process continues.

1.2 Process of Encoding Segmented Image

Fig 6 is a flow diagram of the method 106 of encoding the segmented image utilized in Fig. 1. The input data to the method 106 is a pixel-map representation of the image which has previously been segmented into regions in step 104, each region being defined as a set of connected pixels. In the method 106 a different name is assigned to each segmented region, which can be done arbitrarily. The method 106 comprises two consecutive steps: splitting 604 each region's shape into non-overlapping rectangular blocks; and merging 606 rectangular blocks which belong to the same region in order to limit their number. The step 606 outputs to step 108 a coded representation of the segmented image. The regions of the segmented image are defined as a list of distinct rectangles. Specifically, each region of any arbitrary shape, is partitioned into a set of non-overlapping blocks which cover its entire area. The preferred method 106 limits the number of blocks used for each region. The coded representation is outputted in the form of a set co-ordinates of the vertices of the rectangles together with their associated color information. In this way, the storage requirements for the segmented image are minimized. Moreover, since it is faster to iterate through a list of blocks than through a list of pixels, the access to the pixels of the region for feature computation is faster. The preferred process 604 for splitting the segmented image is described in more detail in the section herein entitled "*1.2.1 Process of Splitting the Segmented Image*". The preferred process 606 for merging the split regions is described in more detail in the section herein entitled "*1.2.2 Process for Merging the Split Regions*".

1.2.1 Process of Splitting the Segmented Image

The splitting step is based on a quad-tree splitting approach. The input image, i.e. the initial block, is divided into 4 rectangular subblocks and each block is recursively split into 4 subblocks until the blocks are included (or mostly included) in a region or reach a minimum size. This implies finding all the regions which are represented in each block and the area of overlapping between the block and those regions. This search can be limited by using the following scheme.

Beforehand, all the region names are stored in a list. The initial list of all the names is associated with the initial block. Then a reduced list of names will be associated with each block, which consists of the names of the regions which can be found in the block; those names are the names of the regions which were previously found in the parent block. Thus, the search of the regions which are present in a given block is not too long.

The following pseudo-code is illustrative of the process 604 of splitting the segmented image into regions as used in Fig. 6.

pseudocode SPLIT

Variables

MIN_SIZE	CONSTANT: Minimum size for rectangles (e.g. minimum height and width).
5 DELTA	CONSTANT: If more than DELTA percent of a rectangle is covered by a single region, then the rectangle is considered to be included in the region (DELTA is a percentage between 0.5 and 1).
NO_REGIONS	Constant: Number of regions in the image.
RECT	A rectangle.
10 NAME_LIST	A list of region names.
RECT_QUEUE	FIFO queue containing rectangles to be processed.
NAME_QUEUE	FIFO queue containing the lists of possible region names for each rectangle (i.e. the names of the regions which can be found in the rectangle) in RECT_QUEUE.
15 REGION[]	OUTPUT: An array of lists of rectangles, of length NO_REGIONS. Each list of rectangles corresponds to the rectangles which are included in a specific region of the image. (The regions are ordered arbitrarily.)

20 *Procedure*

Initialise RECT_QUEUE with the rectangle corresponding to the whole image and NAME_QUEUE with the list of all the region names.

while (RECT_QUEUE is not empty)

25 remove first element from RECT_QUEUE and keep it in RECT
 remove first element from NAME_QUEUE and keep it in NAME_LIST
 if (RECT is bigger than MIN_SIZE)
 if (there is a region of NAME_LIST which takes part in, at least,
 DELTA of RECT)
30 add RECT to REGION[index of that dominant region's name]
 else split RECT, add the four new rectangles into RECT_QUEUE,
 and add the new name list - made of the name of the regions which
 actually were in RECT - in NAME_QUEUE.
 else find the dominant region in RECT and add RECT to REGION[index of
35 the region's name]

Turning now to Fig. 7, there is shown a flow chart of the last mentioned pseudo code called SPLIT for splitting the segmented image. The processing commences at step 104. In the next step 704, the entire image is stored in a FIFO

queue called RECT-queue for processing. In addition, the names of the regions which can be found in the current rectangle are stored in a FIFO queue called NAME_QUEUE. After step 706, the processing continues at decision box 708.

5 In decision box 708, a check is made whether the FIFO queue called RECT_QUEUE is empty. If the decision box returns true, then the processing continues at step 710 where the splitting process terminates and continues with the merging process 606. If the decision box 708 returns false, the processing continues at step 710. In step 710 a variable called RECT is set to the first rectangle in the RECT_QUEUE. Subsequently this rectangle is then removed from the queue called
10 RECT_QUEUE. In the next step 712 the first name in the NAME_QUEUE is retrieved and stored in the NAME_LIST. The name is then deleted from the NAME_QUEUE. Afterwards the processing continues at decision box 714.

In decision box 714, a check is made whether the size of the rectangle is greater than a minimum size MIN-SIZE. If the decision box 714, returns false then the
15 processing continues to step 716. In step 716, the dominant region within the rectangle is determined and this rectangle is added to a list of rectangles and indexed by the dominant regions name. The dominant region is preferably the largest region within the rectangle. After step 716, the processing returns to decision box 708.

If the decision box 714 returns true, then the processing continues at decision
20 box 718. In decision box 718 a check is made whether there exists a region REGION[i] on the NAME_LIST whose intersection with the rectangle is greater than the value DELTA times the size of the rectangle. It is in this way, the process determines the dominant segmented region within the rectangle. If the decision box 718, returns true then the processing continues at step 720. In step 720, the rectangle
25 is added to the list of rectangles REGION[]. The rectangle is added to the REGION[i] array at that location in the array corresponding to the index of dominant regions name. After step 720 the processing continues at step 722.

If the decision box 718 returns false, then the processing continues at step 722. In step 722, the rectangle is split into four sub-rectangles. These four sub-rectangles
30 are then added to the FIFO queue RECT_QUEUE and the names of the regions which were in the previous rectangle are added to the FIFO queue NAME_QUEUE. After step 722 the processing continues at the decision box 708.

Fig 7B to 7E illustrate an example of the splitting process 604 used in Fig. 6. Initially a segmented image 750 is inputted. The segmented image is in the form a pixel
35 map representation having a number of segmented regions 752, 754, 756, and 758 having the same associated color data. In the first step, the whole segmented image is input as the current rectangle. Next a check is made whether the dominant segmented region, that is the largest segmented region in the current rectangle, is greater than 95%

the current rectangle's size. In the present example, this check reveals that the segmented regions 752, 754, 756 and 758 are less than 95% the size of the whole image. In the next step the process divides the segmented image into four rectangles 760, 762, 764, and 766. The process again makes the same check commencing on
5 rectangle 760. In the present example, the check reveals the dominant region 752 is greater than 95% of the current rectangle size and the vertices of this rectangle are stored against this dominant region 752. The same check is made against rectangle 766, which reveals no dominant region and as such the rectangular is further split into four sub-rectangles 768, 770, 772, and 774. This processing continues until all of split sub-
10 rectangles are allocated to one of the segmented regions 752, 754, 756, and 758 of Fig. 7E. In the event, the splitting continues below a predetermined size MIN_SIZE, the most dominant segmented region in this case is allocated the rectangle. Preferably, MIN_SIZE is greater than a 1x1 pixel block, but not too large either. After the processing is completed, the segmented regions 752, 754, 756, and 758 are each
15 represented by vertices of a series of rectangles.

1.2.2 Process for Merging the Split Regions

The second step 606 of the encoding process 106 consists in merging adjacent rectangles in each region in order to minimise the number of rectangles representing a
20 region. Two rectangles are considered adjacent if they share a common edge; thus, their reunion is still a rectangle.

The rectangle lists are still stored in the array REGION[] which comes from the splitting step 604. Adjacent rectangles to a specific rectangle are found by scanning the rectangle list and picking the first adjacent rectangle found. The pseudocode for
25 performing the preferred merging is shown below.

```
for (i = 0 to i = NO_REGIONS - 1)
    count = 0
    while (count < number of rectangles in REGION[i])
30         if (there is an adjacent rectangle of rectangle count, in region i)
merge both rectangles, add the result in REGION[i] and remove them from the list
        else count++
```

Turning now to Fig. 8A, there is shown a flowchart of the last mentioned pseudo code for merging the rectangles. After the splitting step 604, the processing
35 commences at step 804, where a loop counter called *i* is set to zero. After which processing continues at the decision box 806. In the decision box 806 a check is made whether the loop counter *i* is less than the constant NO_REGIONS-1. If the decision box 806 returns false, the processing terminates at step 808.

If the decision box 806 returns true then the processing continues at step 810 where the variable called COUNT is set to zero, after which the processing continues at decision box 812. In decision box 812, a check is made whether the variable count is less than the number of rectangles in REGION[i]. If the decision box 812 returns false then the processing continues at step 814 where the loop counter i is incremented by one and the processing is returned to decision box 806.

The decision box 812 returns true then the processing continues at the decision box 814. In decision box 814, a check is made whether there exists an rectangle adjacent to the rectangle COUNT in REGION[i]. If the decision box 814 returns false then the processing continues at step 816 where the variable COUNT is incremented by one. After step 816 the processing is returned to decision box 812.

If the decision box 814 returns true then the processing continues at step 818. In step 818 both the adjacent rectangles are merged together and the result is listed in the REGION[i] at the end of the list. In addition, both the rectangles are removed from the list of rectangles stored in REGION[i] in step 820. After step 820, the processing returns to decision box 812.

Turning now to 8B, there is shown an example of merging the split segmented images of Fig. 7E in accordance with the preferred merging process of Fig. 8A. The primary step includes merging the adjacent rectangles within each segmented region 752, 754, 756, and 758. Fig. 8B illustrates the merged adjacent regions. The output for this process is a list of the vertices of rectangles for each of the segmented regions 752, 754, 756, and 758. Thus instead of storing against each segmented region each pixel within the segmented region, the preferred process stores the vertices of a series of rectangles that make up that segmented region.

1.3 Markov Random Field (MRF) Merging Process

The preferred merging process 110 is based on a Markov Random Field model.

In the preferred Markov Random Field model, $X = \{X_i, 1 \leq i \leq N\}$ denotes a set of random variables where X_i is associated with region R_i of the segmented image, and $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_m\}$ is a set of possible labels such that $X_i \in \Lambda$ for all i.

For the MRF segmenter, the labels correspond to region names ($m \leq N$). In the initial segmentation obtained from the initial segmentation 104, the image is assumed to be over-segmented. The MRF segmenter iteratively updates the region names so that similar regions get the same name and are eventually merged.

In this model $\omega = (X_1 = x_1, X_2 = x_2, \dots, X_N = x_N)$ is one of all instances from the configuration space. The vector X is a Markov random field if:

1. $P(\omega) > 0$, for all ω ;

2. $P(X_i = x_i \mid X_j = x_j, R_j \neq R_i) = P(X_i = x_i \mid X_j = x_j, R_j \in G_i)$,
 where $P(\bullet)$ and $P(\bullet \mid \bullet)$ are the joint and conditional probability density functions (pdf)
 respectively, and G_i is the set of all the nodes in R which are neighbors of R_i .
 Intuitively, the MRF is a random field with the property that the statistic at a particular
 5 node depends only on that of its neighbors.

The pdf of the MRF has a general form, known as the Gibbs distribution:

$$P(\omega) = Z^{-1} \exp(-U(\omega)), \quad \text{Eqn (4)}$$

where Z is a normalizing constant and $U(\omega)$ an energy function.

The function $U(\omega)$ is obtained by summing some clique functions $V_c(\omega)$:

$$10 \quad U(\omega) = \sum_{c \in C} V_c(\omega), \quad \text{Eqn (5)}$$

$V_c(\omega)$ is a clique function which depends only on the variables X_i such that the region
 R_i belongs to clique c .

The MRF segmenter is preferably based on a region process, i.e. it merges
 regions based on their similarity. Since there should exist a discontinuity on the
 15 common boundary between two different segmented regions, the MRF segmenter could
 also use a boundary process.

The clique functions are defined based on the constraint that a segmented
 region should be uniform in a set of features such as intensity, color, texture, etc. A list
 of features that can be used is shown in Table B.

20 In this MRF model, F_i^k denotes the k^{th} feature for region R_i and M the
 number of features. Cliques which contain two regions are only considered and the
 clique functions are defined as:

$$V_c(\omega) = \sum_{\{R_i, R_j\} \in C} \eta_{ij} \max_{1 \leq k \leq M} |F_i^k - F_j^k|, \quad \text{Eqn (6)}$$

where η_{ij} is a binary variable which has the value 1 if X_i and X_j have the same region
 25 label, and the value 0 otherwise.

All the features are normalized using the mean and standard deviation
 computed for all the regions of the initial RAG, wherein the mean for each feature is
 zero and the STD is one. This allows the merging method to consider only the most
 discriminant feature for each region pair, as shown in equation (6).

30

Table B. Description of the features measured for a region. $\text{Cov} = 3 \times 3$ covariance
 matrix; lum_x = normalized luminance at pixel x ; N_x = set of 4 neighbors of pixel x .

Feature	Definition
r	Mean $R / (R + G + B)$
g	Mean $G / (R + G + B)$
b	Mean $B / (R + G + B)$
C1	Cov[0,0]
C2	Cov[0,1]
C3	Cov[0,2]
C4	Cov[1,0]
C5	Cov[1,1]
C6	Cov[1,2]
MLum	Mean luminance
SDLum	Standard deviation of luminance
r-b	
r-g	
saturation	$[\max(r, g, b) - \min(r, g, b)] / \max(r, g, b)$
smoothness	$\frac{1}{\text{area}(R_i)} \sum_{x \in R_i} \sum_{y \in N_x} \frac{ \text{lum}_x - \text{lum}_y }{4}$

The segmented image is merged by optimizing of the MRF model. MAP estimators aim at finding the configuration of the random field which maximizes the probability distribution eqn (4), i.e. minimizing the energy function eqn (5).

The minimization method used in the preferred merging process is a deterministic relaxation process based on a “highest confidence first principle”. The binary clique which has the smallest $V_c(\omega)$, i.e. the smallest difference in features, is updated first: one of the regions takes the other region’s name and both regions are merged. Then, the segmenter keeps updating the clique which has the smallest clique function until $V_c(\omega)$ is bigger than a predefined energy threshold T_e . Thus, regions having features which are different stay separated.

Once two regions are merged they can’t be split again. After each merging, the energy needs to be updated only in the vicinity of the two regions which have just been merged.

Fig 9 is a flow chart of the preferred method of merging the regions of the segmented images as used in Fig. 1. After RAGs of the segmented regions have been built by the step 108, the merging process commences. In step 902, clique functions $V_c(\omega)$ for all pairs of neighboring segmented regions. In the next step 904, the pair of segmented regions are selected which has the smallest clique function value. The

processing continues at decision block 906, where a check is made whether this clique function value is less than a predetermined threshold value. If the decision block 906 returns true, then the processing continues at step 908. In step 908, both selected regions are merged and the features for merged regions are recomputed. In addition, the clique functions of the merged region with neighboring regions is updated. The merging process continues until the decision block 906 determines clique functions are greater than or equal to the predetermined threshold value. The merging process then terminates at step 112, where the final segmented image is outputted.

This merging process allows a good trade off between accuracy and speed.

Preferred Embodiment of Apparatus(s)

The image segmentation processes are preferably practiced using a conventional general-purpose computer, such as the one shown in Fig. 10, wherein the processes of Figs. 1 to 9 may be implemented as software executing on the computer. In particular, the steps of the image segmentation method are effected by instructions in the software that are carried out by the computer. The software may be divided into two separate parts; one part for carrying out the image segmentation methods; and another part to manage the user interface between the latter and the user. The software may be stored in a computer readable medium, including the storage devices described below, for example. The software is loaded into the computer from the computer readable medium, and then executed by the computer. A computer readable medium having such software or computer program recorded on it is a computer program product. The use of the computer program product in the computer preferably effects an advantageous apparatus for image segmentation in accordance with the embodiments of the invention.

The computer system 1000 consists of the computer 1002, a video display 1016, and input devices 1018, 1020. In addition, the computer system 1000 can have any of a number of other output devices including line printers, laser printers, plotters, and other reproduction devices connected to the computer 1002. The computer system 1000 can be connected to one or more other computers via a communication interface 1008c using an appropriate communication channel 1030 such as a modem communications path, a computer network, or the like. The computer network may include a local area network (LAN), a wide area network (WAN), an Intranet, and/or the Internet

The computer 1002 itself consists of a central processing unit(s) (simply referred to as a processor hereinafter) 1004, a memory 1006 which may include random access memory (RAM) and read-only memory (ROM), input/output (IO) interfaces 1008a, 1008b & 1008c, a video interface 1010, and one or more storage devices

generally represented by a block 1012 in Fig. 10. The storage device(s) 1012 can consist of one or more of the following: a floppy disc, a hard disc drive, a magneto-optical disc drive, CD-ROM, magnetic tape or any other of a number of non-volatile storage devices well known to those skilled in the art. Each of the components 1004 to 1012 is typically connected to one or more of the other devices via a bus 1014 that in turn can consist of data, address, and control buses.

The video interface 1010 is connected to the video display 1016 and provides video signals from the computer 1002 for display on the video display 1016. User input to operate the computer 1002 can be provided by one or more input devices 1008b. For example, an operator can use the keyboard 1018 and/or a pointing device such as the mouse 1020 to provide input to the computer 1002.

The system 1000 is simply provided for illustrative purposes and other configurations can be employed without departing from the scope and spirit of the invention. Exemplary computers on which the embodiment can be practiced include IBM-PC/ATs or compatibles, one of the Macintosh (TM) family of PCs, Sun Sparcstation (TM), or the like. The foregoing are merely exemplary of the types of computers with which the embodiments of the invention may be practiced. Typically, the processes of the embodiments, described hereinafter, are resident as software or a program recorded on a hard disk drive (generally depicted as block 1012 in Fig. 10) as the computer readable medium, and read and controlled using the processor 1004. Intermediate storage of the program and pixel data and any data fetched from the network may be accomplished using the semiconductor memory 1006, possibly in concert with the hard disk drive 1012.

In some instances, the program may be supplied to the user encoded on a CD-ROM or a floppy disk (both generally depicted by block 1012), or alternatively could be read by the user from the network via a modem device connected to the computer, for example. Still further, the software can also be loaded into the computer system 1000 from other computer readable medium including magnetic tape, a ROM or integrated circuit, a magneto-optical disk, a radio or infra-red transmission channel between the computer and another device, a computer readable card such as a PCMCIA card, and the Internet and Intranets including email transmissions and information recorded on websites and the like. The foregoing are merely exemplary of relevant computer readable mediums. Other computer readable mediums may be practiced without departing from the scope and spirit of the invention.

The methods of image segmentation may alternatively be implemented in dedicated hardware such as one or more integrated circuits performing the functions or sub functions of the image segmentation. Such dedicated hardware may include graphic

processors, digital signal processors, or one or more microprocessors and associated memories.

5 The foregoing only describes a small number of embodiments of the present invention, however, modifications and/or changes can be made thereto by a person skilled in the art without departing from the scope and spirit of the invention. The present embodiments are, therefore, to be considered in all respects to be illustrative and not restrictive.

The following numbered paragraphs set forth aspects of the invention, including

1. A method of segmenting an image including the steps of
 - (i) generating a list of neighboring pixels to a region of pixels;
 - 5 (ii) scanning a limited number of said listed neighboring pixels in a predetermined manner;
 - (iii) determining a value, for each said scanned pixel, indicative of the similarity of the scanned pixel with the region of pixels;
 - (iv) selecting a pixel from the scanned pixels that has a minimum value;
 - 10 (v) adding said selected pixel to said region;
 - (vi) adding new neighbors of said region to said list; and
 - (vii) repeating steps (ii) to (v) until the image is segmented.
2. A method of segmenting an image including the steps of
 - 15 (i) generating a list of neighboring pixels to a region of pixels;
 - (ii) scanning a limited number of said listed neighboring pixels in a predetermined manner;
 - (iii) determining a value, for each said scanned pixel, indicative of the similarity of the scanned pixel with the region of pixels;
 - 20 (iv) selecting the first pixel from the scanned pixels that has a said value below a threshold;
 - (v) adding said selected pixel to said region;
 - (vi) adding new neighbors of said region to said list; and
 - (vii) repeating steps (ii) to (v) until the image is segmented.
- 25 3. An apparatus for implementing the method as set forth in paragraph 1 or 2.
4. A computer program product, including a computer readable medium having recorded thereon a computer program for implementing the method as set forth in
30 paragraph 1 or 2.

DATED this FIRST day of OCTOBER 1998
Canon Kabushiki Kaisha

Patent Attorneys for the Applicant
SPRUSON & FERGUSON

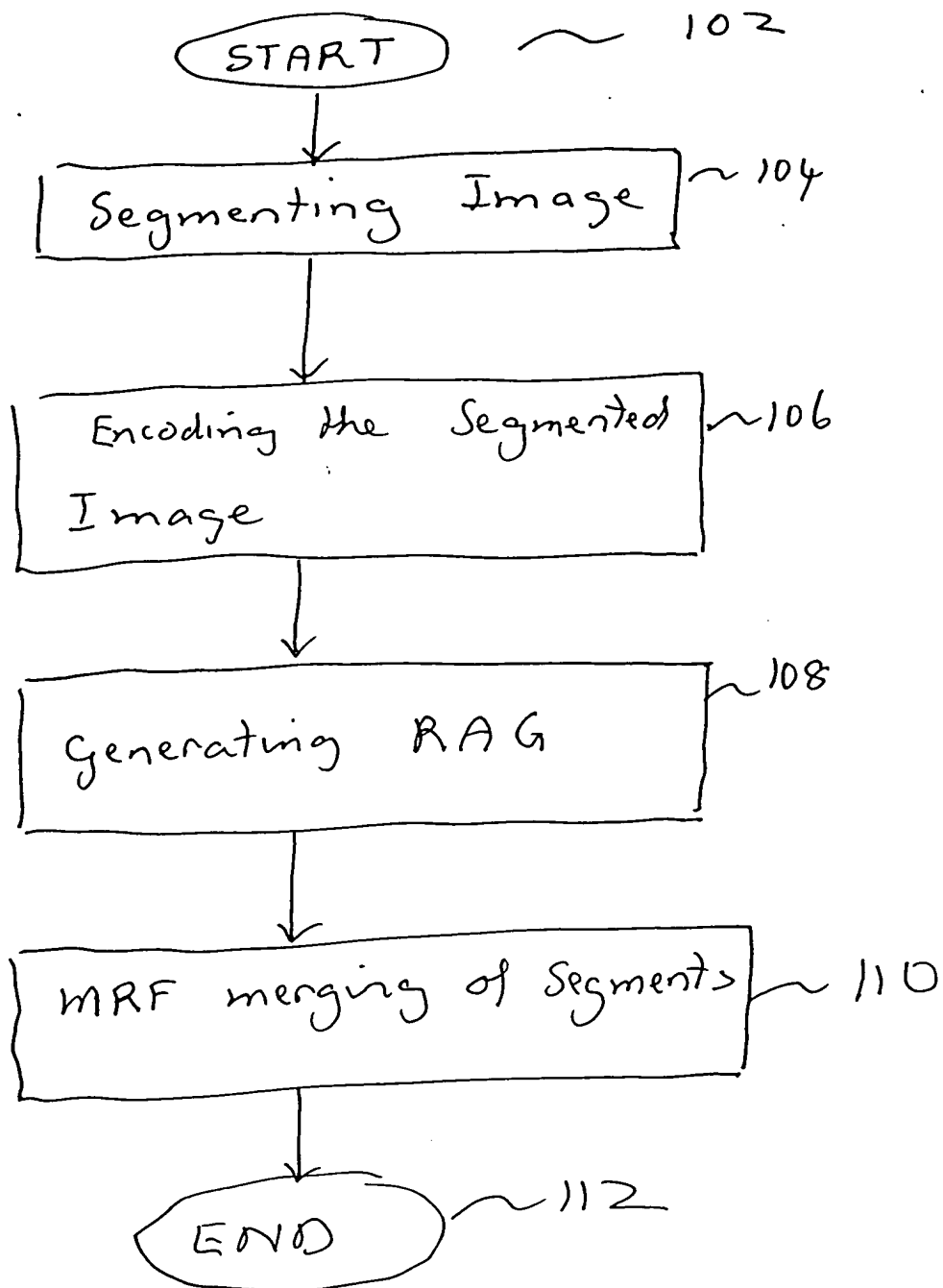


FIG. 1.

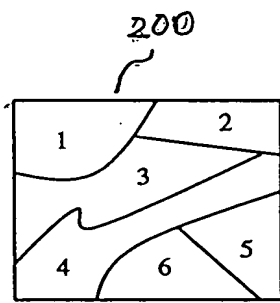


FIG. 2A

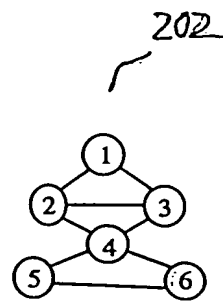
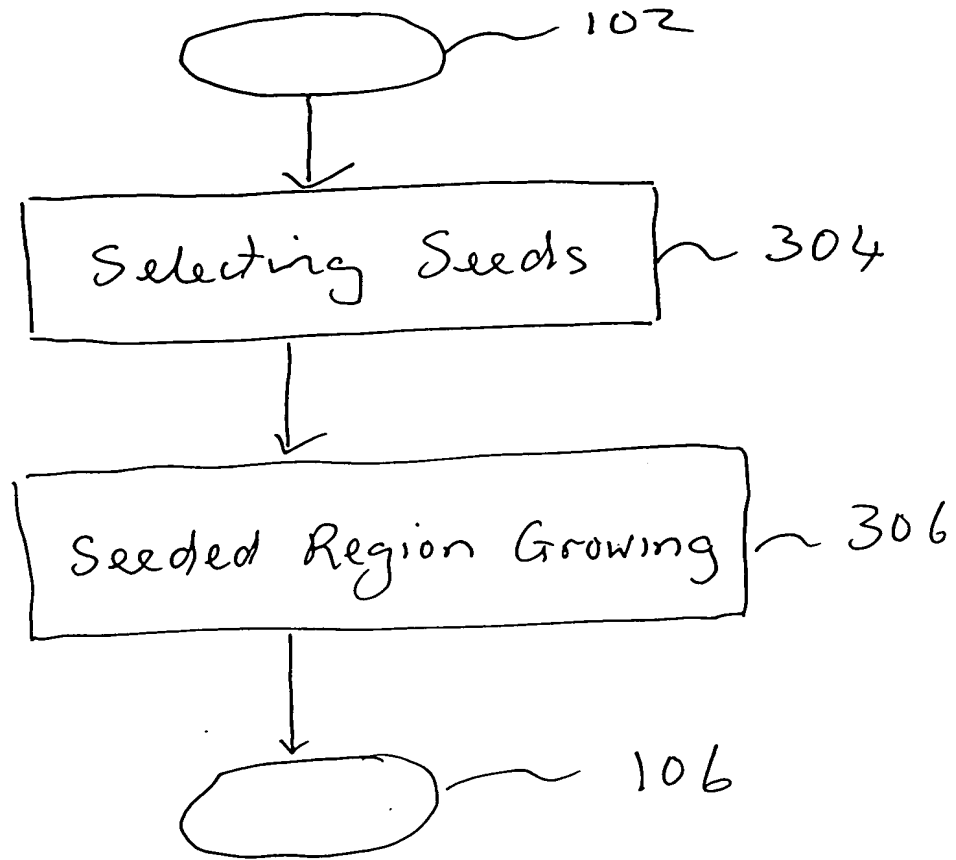


FIG. 2B



2 104

Fig 3

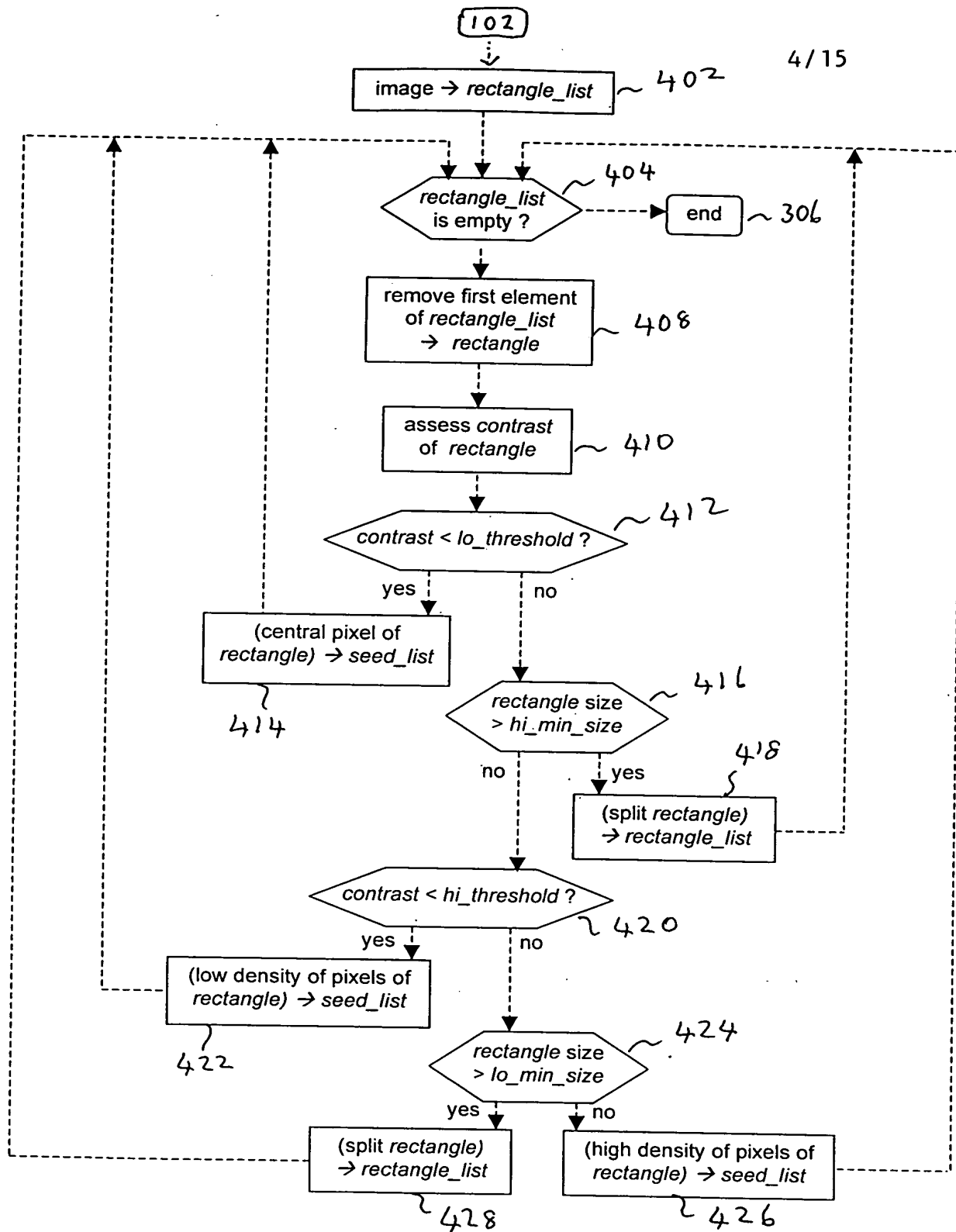


Fig. 4A

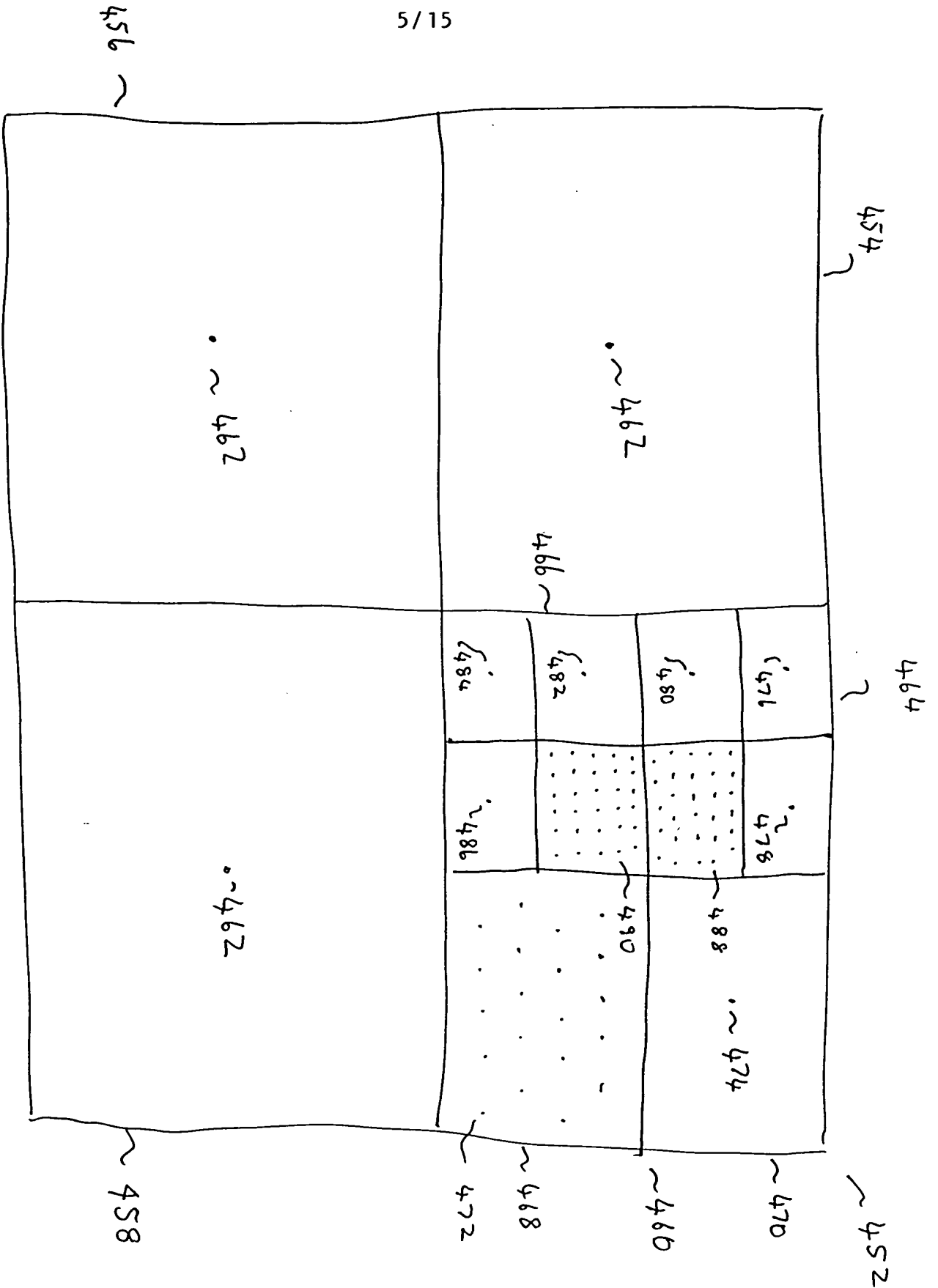
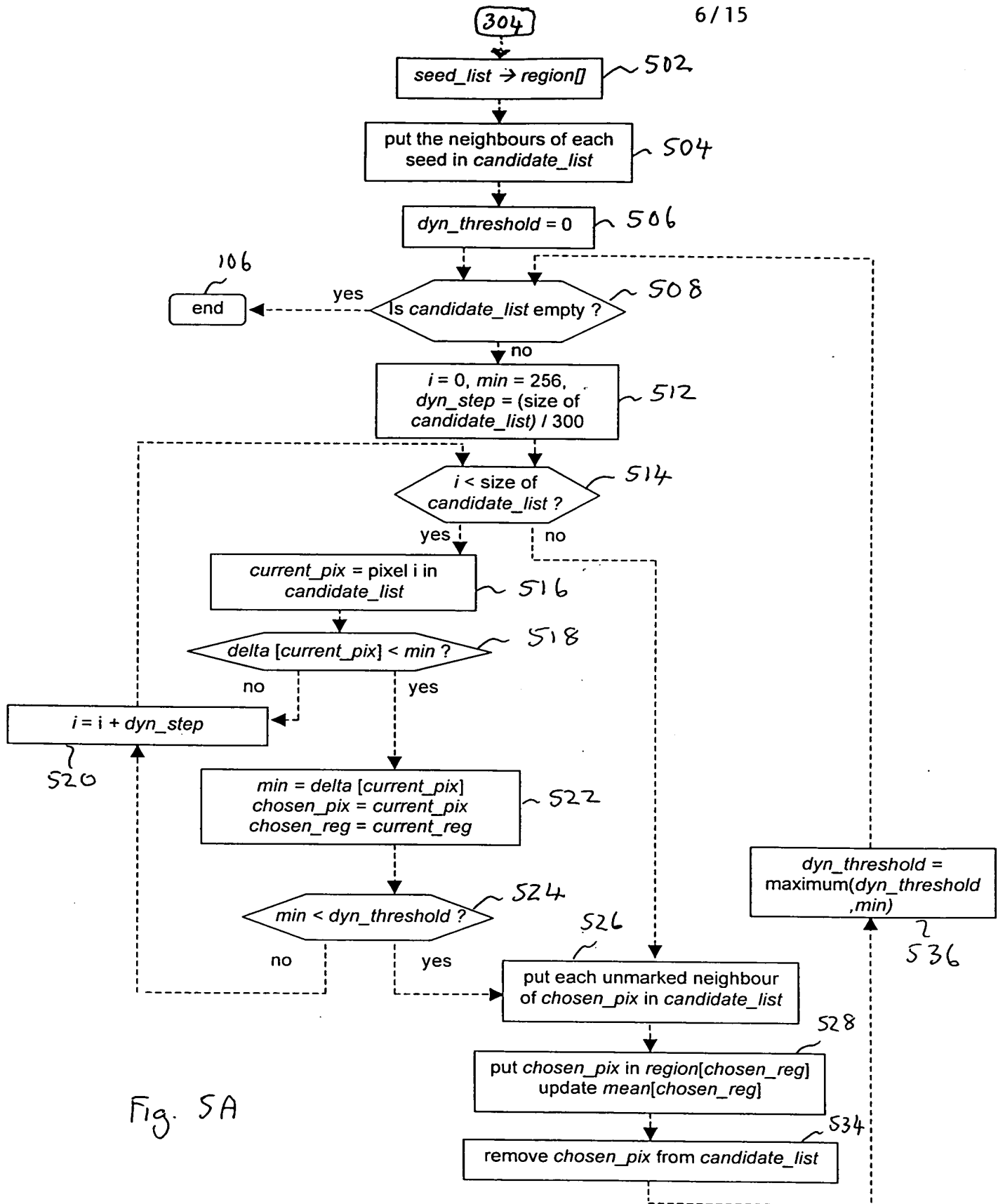


Fig 4B



7/15

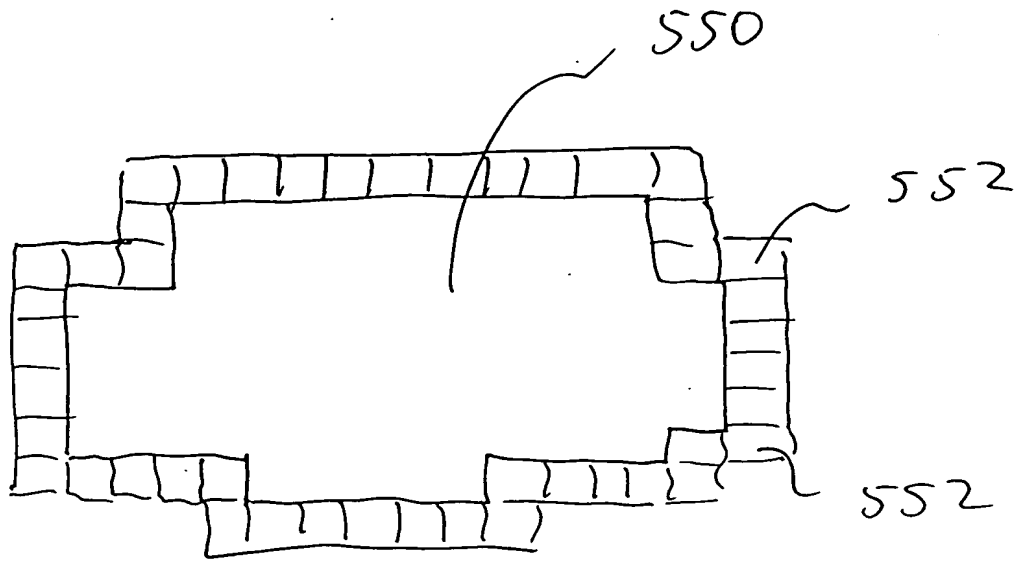


Fig. 5B

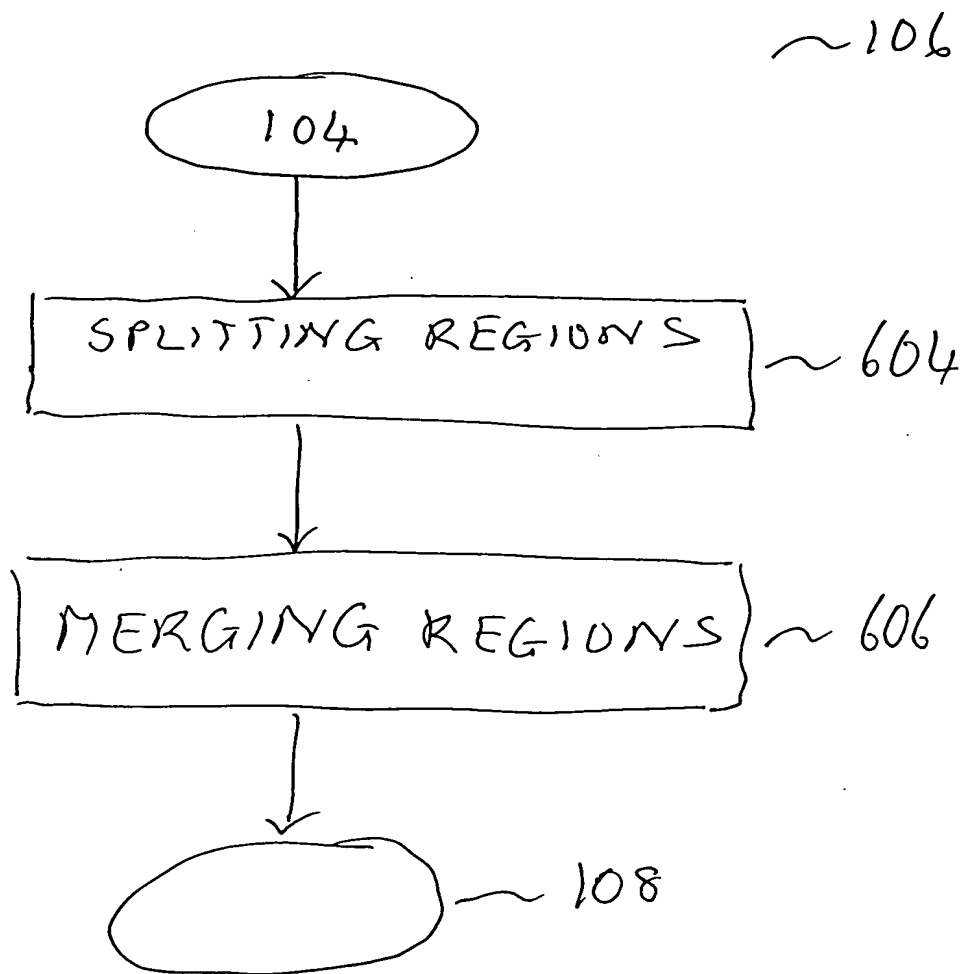


Fig. 6

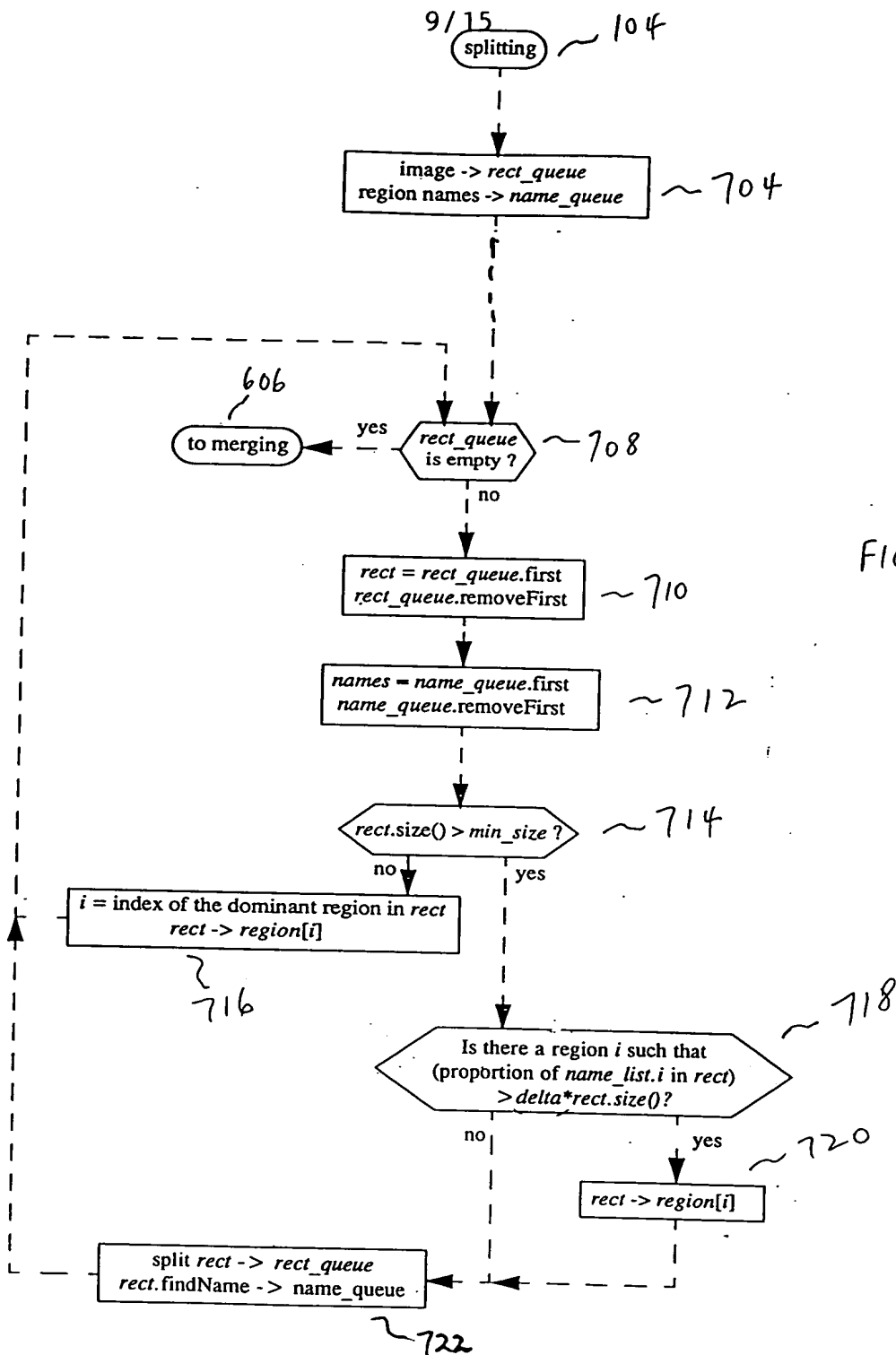


FIG. 7A

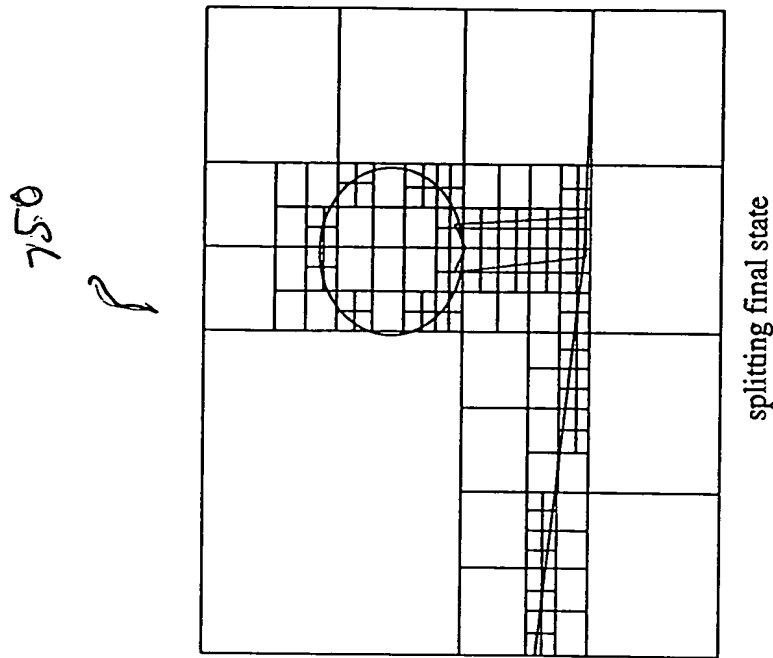


Fig. 7E

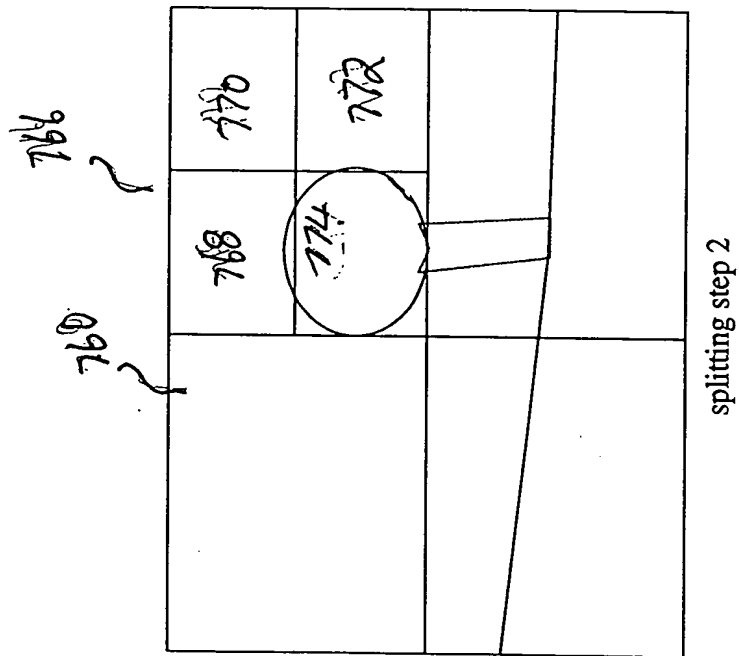
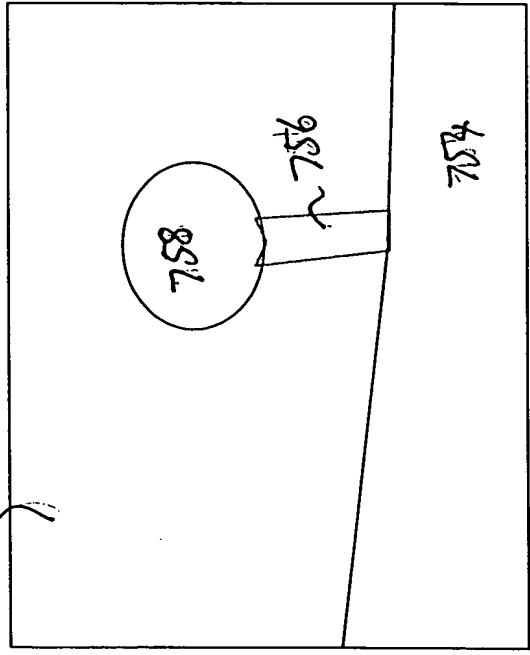


Fig 7D

750

Σ

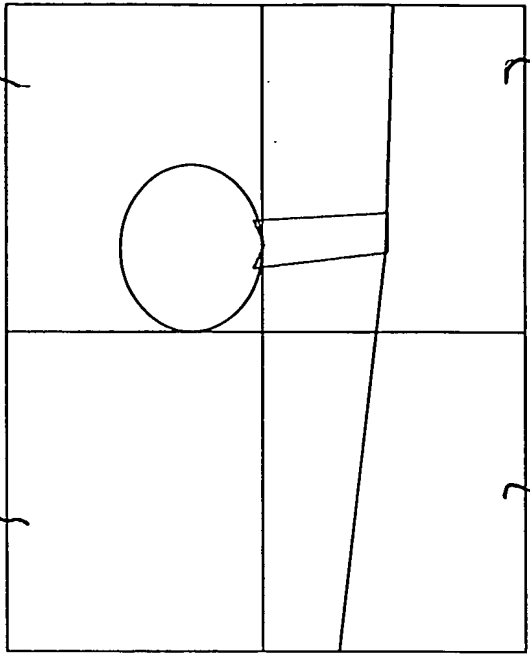
752



splitting initial state

Fig 7B

760 766



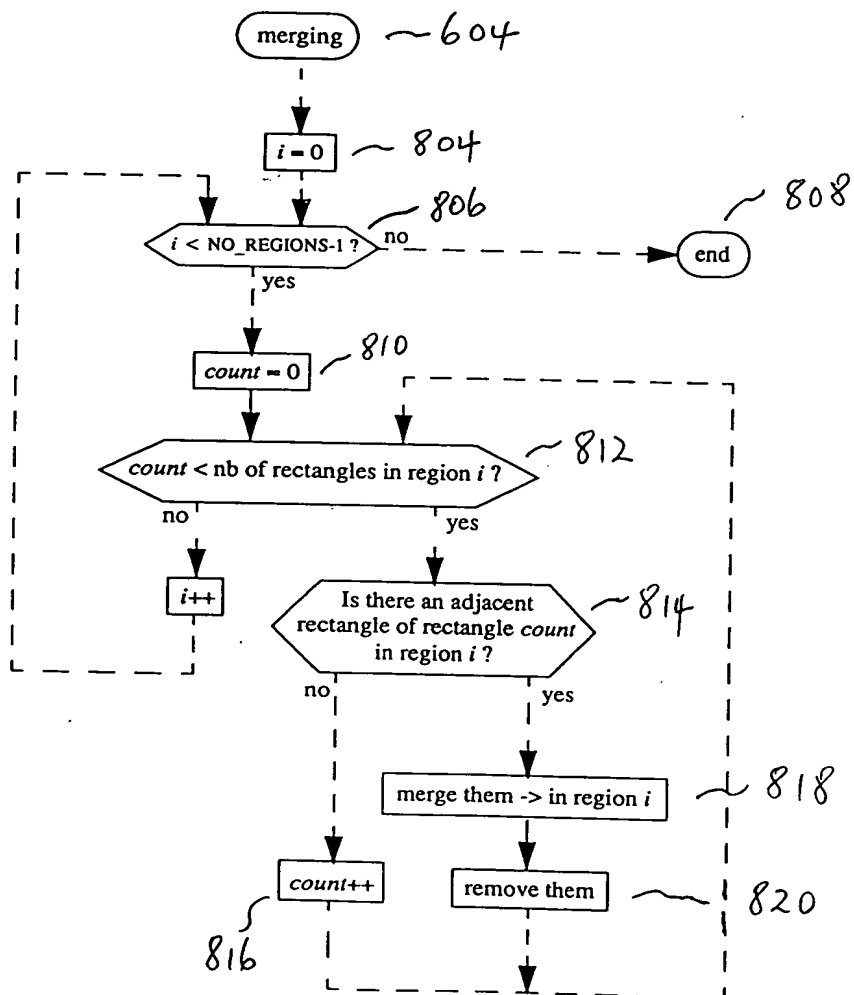
splitting step 1

762

764

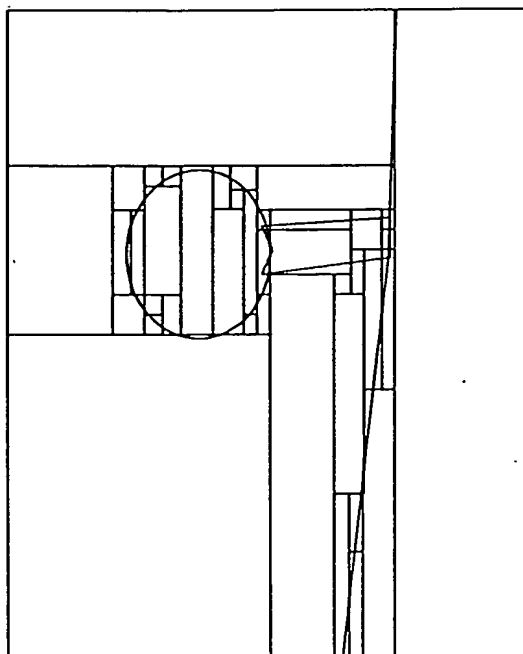
Fig 7C

FIG. 8A



750

~



merging final state

Fig 8B

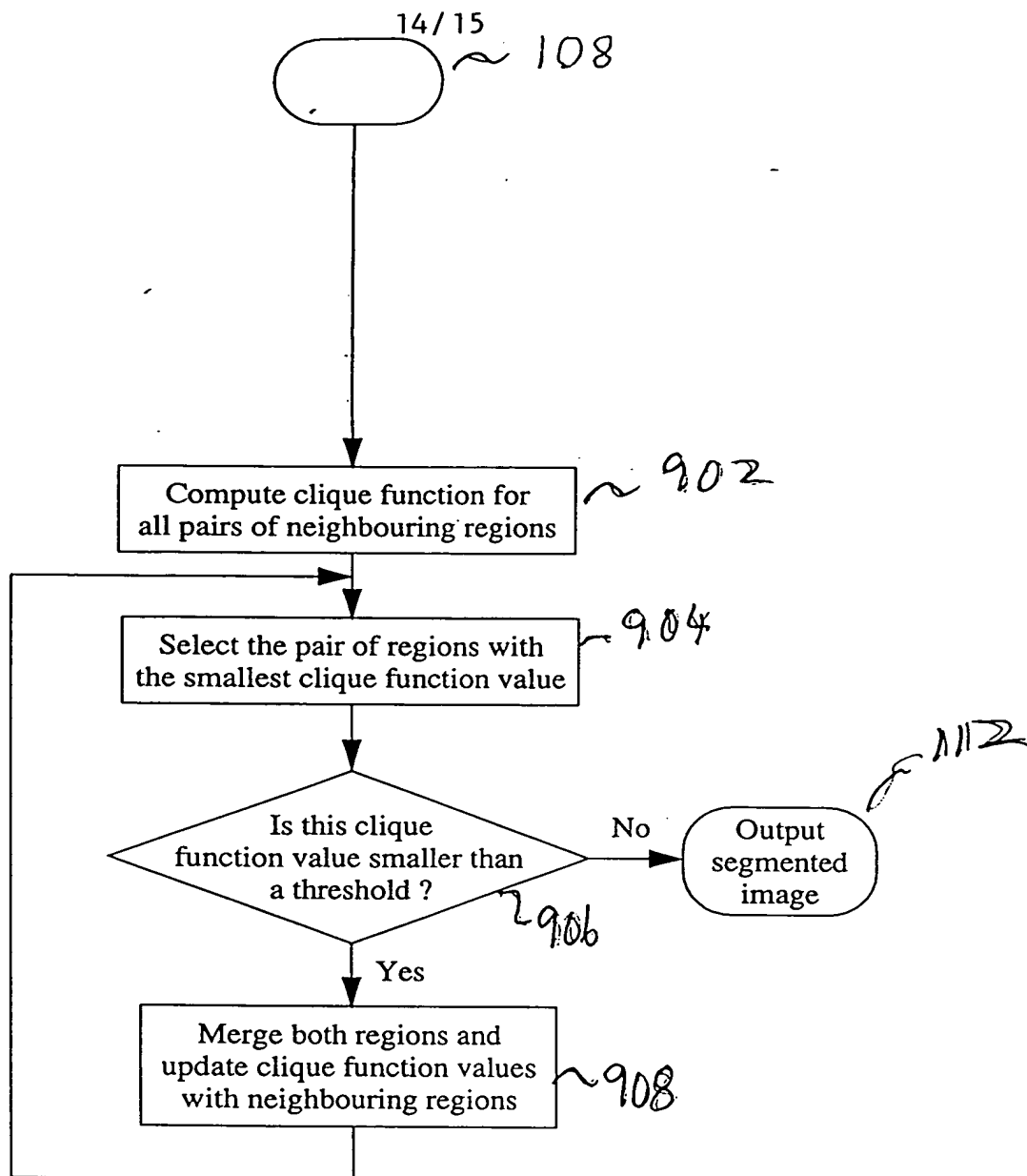
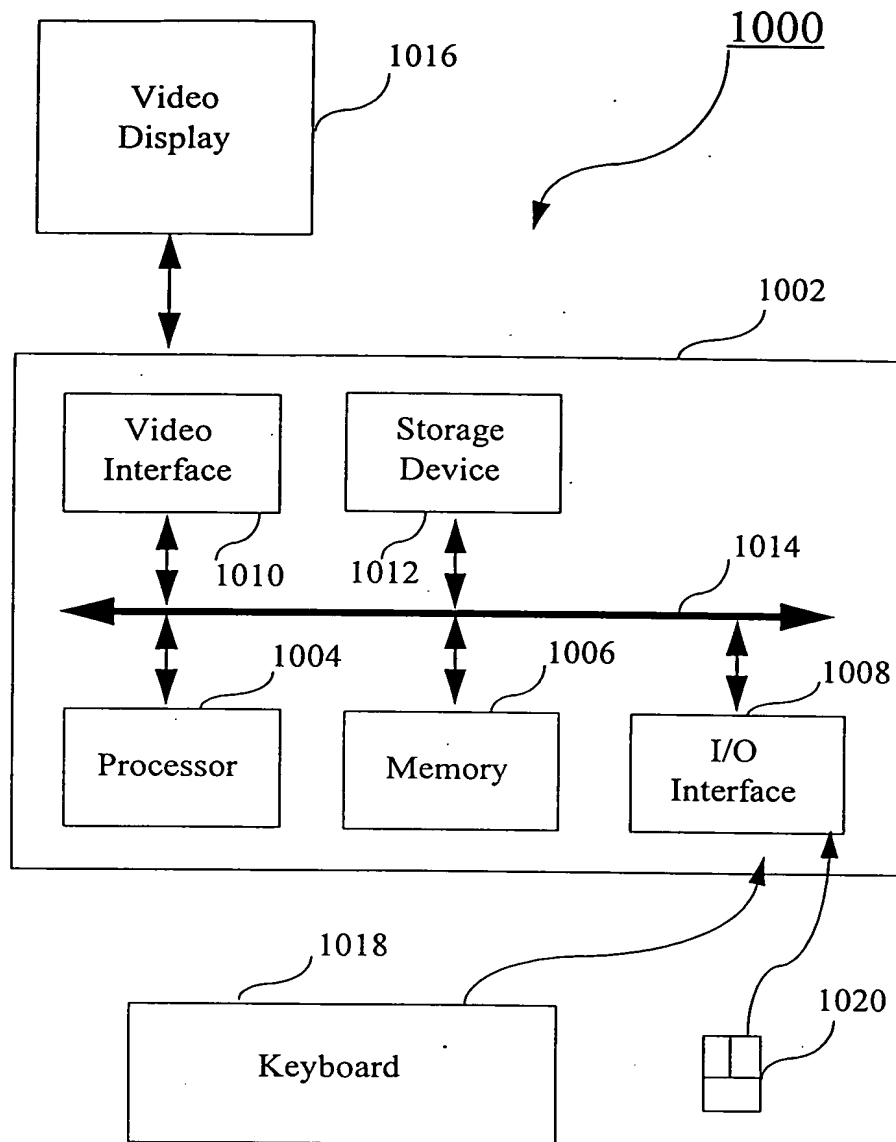


Fig 9

**Fig. 10**